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F. Hirzebruch

Why Do I Like Chern, and Why Do I Like Chern Classes?

In 1949–50 I studied for three semesters at the ETH in Zurich and learned a lot from Heinz Hopf and Beno Eckmann [1], also about Chern classes, their applications, and their relations to Stiefel-Whitney classes ([2], [3], [4], [5]). Chern classes are defined for a complex vector bundle E over a reasonable space X with fiber $\mathbb{C}^n$. They are elements of the cohomology ring of X. The ith Chern class of E is an element of $H^{2i}(X, \mathbb{Z})$ where $0 \le i \le n$ and $c_0 = 1$. They are used for the investigation of fields of r-tuples of sections of the vector bundle, in particular if X is a compact complex manifold and E the tangent bundle of X. Then we have the basic fact: If there exists an r-tuple of sections which are linearly independent in every point of X, then $c_i = 0$ for $i \ge n - r + 1$. For real differentiable manifolds such questions are treated in the dissertation of Hopf's student Stiefel [4], later a well-known computer scientist. For a compact complex manifold X of dimension n, the n-dimensional products of the Chern classes of the tangent bundle (all dimensions complex) give the Chern numbers, when integrated over X, for example $c_n[X]$ is the Euler-Poincaré characteristic (Poincaré-Hopf theorem).

From 1950 to 1952 I was scientific assistant in Erlangen and wrote the paper [6] where ideas of Hopf entered [2]. Some of the results could have been generalized to higher dimensions. But the so-called "duality formula" was not yet proved. This formula says that the total Chern class $1 + c_1 + c_2 + \cdots$ of the direct sum of two complex vector bundles equals the product of the total Chern classes of the summands. The paper [6] has a remark written during proofreading that Chern and Kodaira told me that the "duality formula" is proved in a forthcoming paper of Chern [7]. In the commentary to my paper [6] in volume 1 of my *Collected Papers* (Springer 1987), I write that my knowledge about Chern classes increased with the speed of a flash when I came to Princeton in August 1952 as a member of the Institute for Advanced Study and talked with K. Kodaira, D. C. Spencer, and, a little later, with A. Borel, who told me about his thesis containing his theory about the cohomology of the classifying spaces of compact Lie groups. For the unitary group $U(n)$, this implies that the Chern class c_i can be considered in a natural way as the ith elementary symmetric function in certain variables $x_1, x_2, \ldots, x_n$.

My two years (1952–54) at the Institute for Advanced Study were formative for my mathematical career ([8],[9]). I had to study and develop fundamental properties of Chern classes, introduced the Chern character, which later (joint work with M. F. Atiyah) became a functor from K-theory to rational cohomology. I began to publish my results in 1953. The main theorem is announced in [10]. It concerns the Euler number of a projective algebraic variety V with coefficients in the sheaf of holomorphic sections of a complex analytic vector bundle W over V. Chern classes everywhere! I quote from [10]: "The main theorem expresses this Euler-Poincaré characteristic as a polynomial in the Chern classes of the tangential bundle of V and in the Chern classes of the bundle W."

The Chern classes accompanied me throughout all my mathematical life; for example: In 2009 I gave the annual Oberwolfach lecture about Chern classes [11].

My fiancée joined me in Princeton in November 1952. We married. A "marriage tour" was organized, for which Spencer gave me some support from his Air Force project. I lectured in seven places during this trip, including Chicago, where we met the great master Shiing-Shen Chern and his

F. Hirzebruch is professor emeritus at the Universität Bonn and director emeritus of the Max-Planck-Institut für Mathematik in Bonn. His email address is hirzebruch@mpim-bonn.mpg.de.

charming wife. He was forty-one, I was twenty-five. For me he was a gentleman advanced in age. But all shyness disappeared. He was interested in my progress in Princeton about which I also talked in my lecture. We must have spoken about his papers [3] and [7]. Chern begins in [3] with a study of the Grassmannian $H(n,N)$ of linear subspaces of dimension n in the complex vector space of dimension $n+N$. He defines the Chern classes of the n-dimension tautological bundle over the Grassmannian in terms of Schubert calculus. From here Chern comes to the definition using r-tuples of sections. For $N \to \infty$, the Grassmannian becomes the classifying space of $U(n)$, and we are close to what I learned from Borel. For Hermitian manifolds Chern shows how to represent the Chern classes by differential forms.

The paper [7] has the following definition of Chern classes: Let E be a complex vector bundle of dimension n over the base B. Let P be the associated projective bundle with fiber $\mathbb{P}_{n-1}(\mathbb{C})$. Let L be the tautological line bundle over P and $g = -c_1(L)$. Then g restricted to the fiber of P is the positive generator of $H^2(P_{n-1}(\mathbb{C}), \mathbb{Z})$. Integration of g^{n-1+m} over the fiber in P gives $\bar{c}_m$, the mth "dual" Chern class of E. The total "dual" Chern class $\bar{c} = 1 + \bar{c}_1 + \bar{c}_2 + \cdots$ is defined by

$$c \cdot \bar{c} = 1.$$

If $B = H(n,N)$, then $\bar{c}$ is the total Chern class of the complementary N-dimensional tautological bundle over B.

Chern uses this to prove that the Chern classes are represented by algebraic cycles if everything happens in the projective algebraic category.

The Cherns invited my wife and me for dinner in their home. For the first time we enjoyed the cooking of Mrs. Chern. Many meals in Berkeley would follow. The Chern family, with their two children in 1950, can be seen in the photograph on page XX of his *Selected Papers* (Springer 1978). Chern presented me a copy of this book with the dedication "To Fritz. Warmest regards. June 1979". The signature is in Chinese characters.

During 1955–56 I was an assistant professor at Princeton University. I gave a course on my book [12]. Chern and Serre attended at least occasionally. Chern, Serre, and I wrote a paper, "On the index of a fibered manifold", which was submitted in September 1956 [13]. There the multiplicativity of the signature [= index] is proved for fibrations of compact connected oriented manifolds provided the fundamental group of the base acts trivially on the rational cohomology of the fiber.

In 1960 Chern became a professor in Berkeley. I visited him there in 1962, 1963, 1967, 1968, 1973, 1974, 1979, 1983, 1986, and 1998, always with part of my family. Chern inspired an official offer to me by the University of California (November 1968). He wrote to me: "We all hope that you will find Berkeley sufficiently attractive to deserve your serious consideration. Some disturbances are expected but they need not concern you. I am going to submit to the NSF a new proposal for research support and will be glad to include you in the proposal." In Bonn I was very involved in discussions with the protesting students and expected to have a quieter life in Berkeley as a new faculty member with more time for mathematics. Finally I decided to stay in Bonn. Chern was very disappointed. But the invitations to Berkeley continued. The Cherns were always very helpful in many practical problems: picking us up at the airport, finding a house, lending us things useful for housekeeping, even lending us a car, depositing items in their house we had bought to be used during the next visit....We enjoyed the Cherns' hospitality in their beautiful home in El Cerrito, overlooking the Bay with the famous Bay Bridge, or in excellent Chinese restaurants in Berkeley and Oakland where the Cherns were highly respected guests. There were always interesting conversations with the Cherns and the other dinner guests.

In 1979 there was a conference, "The Chern Symposium", on the occasion of Chern's retirement as a professor of the university. In the *Proceedings* [14] I. M. Singer writes: "The conference also reflected Professor Chern's personality, active yet relaxed, mixed with gentleness and good humor. We wish him good health, a long life, happiness, and a continuation of his extraordinary deep and original contributions to mathematics." This came also from my heart.

Chern did not really retire. In 1981 he became the first director of the Mathematical Sciences Research Institute in Berkeley. When the MSRI building was ready, I sometimes used Chern's beautiful office with a wonderful view.

In 1981 I nominated Chern for the "Alexander von Humboldt-Preis". He received it and spent part of the summers of 1982 and 1984 in Bonn. He talked at the Arbeitstagungen of these years on the topics "web geometry" and "some applications of the method of moving frames".

In 1998 I was invited to be one of the first Chern professors in Berkeley. These visiting professorships are financed by Robert G. Uomini, a former student of Chern, who had won an enormous sum in the lottery. In my case a one-day Chern symposium was held, followed by a four-week course. The title of my Chern lecture in the symposium was "Why do I like Chern classes?" I gave four answers:

(1) *The Chern classes remind me of my youth.* I hope this became clear in the beginning of this contribution.

(2) *The Chern classes have so many different definitions.* As a joke I added: *I especially like that all these definitions are equivalent.* There are the definitions in Chern's papers [3] and [7]. The statement in the joke

> needed some work, which was carried out by Borel and me and perhaps by others, too. The difficulty consisted in sign questions: Are we dealing with a complex vector bundle V or its dual V^*?

(3) *"Chern has a beautiful character."*
There was the story that during a lecture about K-theory and its functor ch to rational cohomology I cried out, "Chern has a beautiful character!" Chern was present and smiled.

(4) *Chern classes have so many applications.*

In 1998 Chern was eighty-seven years old. He did not appear so old to me. He came to my Chern lecture and also to some lectures in my four-week course. The Cherns came to an official dinner. They invited us to a Chinese restaurant.

Berkeley 1998! The last time I saw Chern. The Cherns gradually moved to China.

But we stayed in contact. We edited the two volumes on the Wolf Prize winners in mathematics (published by World Scientific in 2000 and 2001).

My retirement as director of the Max Planck Institute for Mathematics in Bonn in 1995 was celebrated by a "party" with informal lectures, performances, music, lunches and dinners organized by Don Zagier. It lasted two or three days. Zagier had the idea to produce a book with essays or short statements by the participants and by some other people who could not attend. Chern did not come. But one page is by him (see Figure 1).

In 2005 the School of Mathematics of the Institute for Advanced Study in Princeton had its seventy-fifth anniversary. Of the older members Chern, Bott, Hirzebruch, and Atiyah were invited to present to the inner circle how the time at the Institute was formative for their careers, Chern by television. But he died in 2004. I also gave a mathematical lecture in which Borel and Chern figured prominently. Chern classes everywhere! Borel and I had shown in the 1950s how to calculate the Chern classes and the Chern numbers of compact complex homogeneous spaces. An example (in a formulation by E. Calabi):

> Let X be the projective contravariant tangent bundle of $\mathbb{P}_3(\mathbb{C})$ and Y the projective covariant tangent bundle. Then the Chern number c_1^5 of these five-dimensional complex homogeneous spaces X and Y, respectively, equals 4500 and 4860. This is interesting because X and Y are diffeomorphic (compare [11] and the work of D. Kotschick mentioned there).

Remark. It is unavoidable that this contribution has some overlap with [15] and with my interview about Chern of December 6, 2010, here in Bonn [Zala Films with George Csicsery for MSRI].

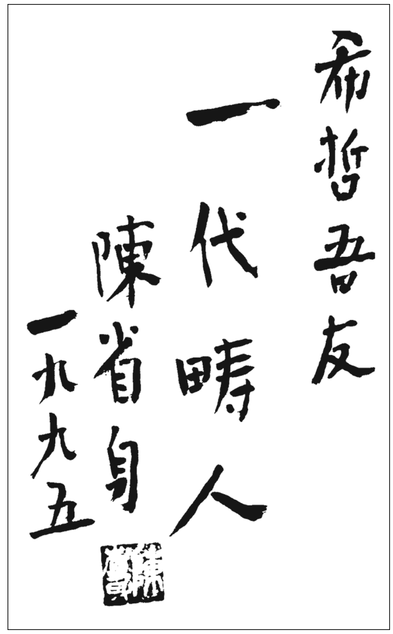

Figure 1.

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Michael Atiyah

Memories of S.-S. Chern

I first met Chern in Chicago in 1956. I had gone to the Institute for Advanced Study in Princeton after my Ph.D., and Chern invited me to give a seminar. He was a senior professor and I a raw Ph.D., but he took good care of my wife and me for our week in Chicago. We remained in frequent touch over the subsequent years, and the last time we saw him was as his houseguests on the campus of Nankai University, shortly before his death. One clear memory I have of him is at a conference in Durham, England, where, despite advancing years, he valiantly walked along Hadrian's Wall with the younger generation.

Chern was a geometer of the old school. His work had none of the polish of the postwar generation, his methods were direct and intuitive and at times cumbersome. For this reason I and others of my generation underestimated him. What he may have lacked in elegance he made up for by his breadth of interest and his deep geometrical insight. This took him in many pioneering directions and led to his extensive collaborations with diverse mathematicians such as Moser, Bott, Simons, and Griffiths. His connection with physicists such as C. N. Yang and T. D. Lee paved the way for the remarkable interaction between geometry and physics of the past few decades.

He was of course a legendary figure in China (and in Chinese restaurants in Berkeley), and it was through him that I and many other mathematicians were introduced to China. The Chern Institute at Nankai is a lasting tribute to his role in revitalizing Chinese mathematics.

I also owe Chern a debt of gratitude for persuading me, at an early stage, to publish my collected works and to make them available in China. As someone who has not fully adapted to modern technology, I find books more friendly and accessible than the electronic media.

Chern's influence, and the widespread affection felt for him by colleagues of all ages, is due in no small part to his personality. Despite becoming the grand old man of Chinese mathematics, he remained modest and unassuming, always willing to listen and to encourage the young. His photograph is one of the few in my study. Alongside it is a framed Chinese poem in beautiful calligraphy that Chern composed on the plane that flew him to England in 1976 for the joint LMS/AMS bicentennial meeting. Since I was LMS president at the time he presented it to me, together with an English translation, discreetly placed on the back.

Michael Atiyah is honorary professor of mathematics at the University of Edinburgh. His email address is `m.atiyah@ed.ac.uk`.

Manfredo do Carmo

On Collaborating with Chern

Chern was probably the most important influence of my life as a mathematician. As time goes by, I find myself using more and more in my work what I learned from him during the times I stayed in Berkeley, first as his student and later as a postdoctoral fellow. He was not a forceful person, and his teachings had to be found in his almost casual remarks and mostly in his personality that was, in a mysterious way, very kind but very firm.

I have already written somewhere else [1] my reminiscences as Chern's student; I now want to make some comments on the experience of collaborating with Chern.

In the winter of 1968 Chern gave a course on a preprint by Jim Simons, "Minimal varieties in Riemannian manifolds", later published in [2]. The paper was a breakthrough in the theory of minimal surfaces, and Chern decided to present the subject from the beginning using the method of moving frames; he worked miracles with this method, and it was beautiful to see how things would develop in a natural way through his treatment. For me, the course was an important opening. I had a secret love for the theory of minimal surfaces, but I had not been able to form a clear view of the subject. But then, sometime along the course, I began to feel at home with the beauty of the topic. One characteristic of any Chern course was the presence of interesting open problems, and this course was no exception. Implicit in Simons's paper was a question that Chern made explicit and proposed as a problem in one of the lectures of the course. I had followed the course closely, and that particular problem attracted me. I worked hard and found a solution that I sketched before the following class. After the class, I approached Chern to show my solution. From the other side,

Manfredo do Carmo is titular researcher at Instituto Nacional de Matemática Pura e Aplicada (IMPA). His email address is `manfredo@impa.br`.